Estimation and Application of Bayesian Hawkes Process Models

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Data: Events





Hawkes Processes

They are used when the occurrence of one event INCREASES / DECREASES

the probability of another events happening after.

Introduced by Alan G. Hawkes in Biometrika in 1971 they more recently gained traction due to their applicability.



Paper citations per year.

Applications

- Earthquakes
- <u>Neural Activity</u>
- Epidemics
- Tradings
- Order Books
- Financial News

- Terrorism
- Crime
- Gun Violence
- Cybersecurity
- Twitter
- Emails
- Memes
- Group Chats

- Product Sales
- Car Accidents
- Power Failure
- Solar Flares
- Bat Migration
- ...

Product Cannibalisation



UNIVARIATE ——

ABC

EXCITATION ONLY EXCITATION + INHIBITION

MULTIVARIATE -





ABC for Hawkes Processes with Missing Event Times

CHAPTER 3

STARTING POINT

Missing data is an issue when working with real-life data from Hawkes Processes.

Classic estimation approaches are biased or inapplicable, as the full model **likelihood is intractable**.

CONTRIBUTION

We use a **simulation-based inference** approach that do not require evaluating the likelihood.

Our method utilises Approximate Bayesian Computation with tailor-made summary statistics.

	RESULI
d	We obtain approximate
Des	posterior samples from our
	proposed ABC-Hawkes
	approach.
	These can adequately
	recover the posterior
ו	distribution for different
	missingness scenarios.

Hawkes Processes with Inhibition

CHAPTER 4

STARTING POINT

When allowing inhibition in a Hawkes Process model, two challenges arise:

- 1. ensuring a **non-negative** intensity
- 2. integrating the intensity

Conditions to check for **stability** under inhibition are unnecessarily strict.

CONTRIBUTION

We use a link function to keep the likelihood nonnegative. Based on this, we provide exact and approximate ways to integrate the intensity.

We introduce a new, **less** restrictive condition for stability.

RESULT

We have clarified intricacies under inhibition. These models become more **usable**.

One unified stability criterion allows for more parameters to be classified as stable.

Hawkes Processes for Product Cannibalisation

CHAPTER 5

STARTING POINT

Understanding product cannibalisation is an **important aspect** of retail analytics. Most approaches consider **cumulative sales** numbers. They only look at **retail** data.

Some priors are dependent on the **dimensionality** of the problem.

CONTRIBUTION

We use a Hawkes Process model, where **inhibition** is interpreted as product cannibalisation. We acknowledge the **temporal** structure of sales. We use **wholesale** data.

We reparametrise the model to introduce **dimension**-**independent priors**.

RESULT

Hawkes process with inhibition provide a way to **uncover product cannibalisation** in wholesale data. They outperform benchmark models without inhibition.

The proposed priors are **useful** for any Hawkes Process application.

Ancestor Hawkes Model

CHAPTER 6

STARTING POINT

Hawkes Processes treat immigrant and triggered events the **same**. This might not be appropriate for all applications.

Little statistical research has been published on **group chat** data.

CONTRIBUTION

We propose the Ancestor Hawkes mode, where immigrant and triggered events can have **different influences**.

We **collect** a group chat data set and use the Ancestor Hawkes model on it.

RESULT

The proposed models are **versatile** tools that can be appropriate for specific applications.

The Ancestor Hawkes captures characteristics in messaging dynamics that the classic Hawkes Process cannot portrait.



Enable and improve estimation. Showcase novel applications.